NUMBER SENSE MAGIC

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This workbook will unveil the magic of Number Sense. It can be used to train students for academic competition. Although younger students may benefit from some of its content, I would recommend that it be used with any student following the fifth grade in Elementary School. Since being introduced to Number Sense in December 1966, I have always been fascinated by the power of the brain. I am confident that you will experience the magic of Number Sense as you work through this workbook. With this workbook, I tried to unveil the beauty of mathematics. I know that learning Number Sense will have a positive impact for years to come.

As always, my wife and family were my inspiration. They have always been supportive of the work that I do in education. They continuously encourage me to find new ways to bring the joy I have experience with Number Sense to others. I sincerely hope that you are touched by the magic of Number Sense.

USING PERCENTS TO APPROXIMATE MULTIPLICATION AND DIVISION

Memorization of the following can greatly improve a student's ability to approximate multiplication and division. Students should begin by memorizing the percents for $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$, etc.

PERCENTS

$\frac{1}{2} = 50\%$ $\frac{1}{3} =$ ~~$66\frac{2}{3}\%$~~ $33\frac{1}{3}\%$ $\frac{2}{3} = 66\frac{2}{3}\%$

$\frac{1}{4} = 25\%$ $\frac{3}{4} = 75\%$ $\frac{1}{5} = 20\%$

$\frac{2}{5} = 40\%$ $\frac{3}{5} = 60\%$ $\frac{4}{5} = 80\%$

$\frac{1}{6} = 16\frac{2}{3}\%$ $\frac{5}{6} = 83\frac{1}{3}\%$ $\frac{1}{7} = 14\frac{2}{7}\%$

$\frac{2}{7} = 28\frac{4}{7}\%$ $\frac{1}{8} = 12\frac{1}{2}\%$ $\frac{3}{8} = 37\frac{1}{2}\%$

$\frac{5}{8} = 62\frac{1}{2}\%$ $\frac{7}{8} = 87\frac{1}{2}\%$ $\frac{1}{9} = 11\frac{1}{9}\%$

$\frac{2}{9} = 22\frac{2}{9}\%$ $\frac{4}{9} = 44\frac{4}{9}\%$ $\frac{1}{10} = 10\%$

$\frac{3}{10} = 30\%$ $\frac{7}{10} = 70\%$ $\frac{1}{11} = 9\frac{1}{11}\%$

$\frac{2}{11} = 18\frac{2}{11}\%$ $\frac{1}{12} = 8\frac{1}{3}\%$ $\frac{5}{12} = 41\frac{2}{3}\%$

$\frac{7}{12} = 58\frac{1}{3}\%$ $\frac{1}{13} = 7\frac{9}{13}\%$ $\frac{1}{14} = 7\frac{1}{7}\%$

$\frac{3}{14} = 21\frac{3}{7}\%$ $\frac{1}{15} = 6\frac{2}{3}\%q$ $\frac{1}{16} = 6\frac{1}{4}\%$

$\frac{3}{16} = 18\frac{3}{4}\%$ $\frac{5}{16} = 31\frac{1}{4}\%$ $\frac{7}{16} = 43\frac{3}{4}\%$

$\frac{1}{18} = 5\frac{5}{9}\%$ $\frac{1}{20} = 5\%$ $\frac{1}{30} = 3\frac{1}{3}\%$

$\frac{1}{40} = 2\frac{1}{2}\%$

Using percents to approximate multiplication and division will give a more accurate approximation.

Approximate each of the following :

Example A : 33 x 93 =

Note : 33 is approximately equal to $33\frac{1}{3}\%$ which is equal to $\frac{1}{3}$. To approximate 33 x 93 first find $\frac{1}{3}$ of 93 which is equal to 31. Since 30 x 90 = 2,700, the more accurate approximation is 3,100. The exact answer is 3069.

Example B : $24 \times 85 =$

Note : 24 is approximately equal to 25% which is equal to $\frac{1}{2}$. To approximate 24 x 85 find $\frac{1}{4}$ of 84 which is equal to 21. Since 20 x 90 = 18,00, the more accurate approximation is 21,200. The exact answer is 2040.

Example C : $167 \times 241 =$

Note : 167 can be thought of as being 16 $\frac{2}{3}$% which is equal to $\frac{1}{6}$. To approximate 167 x 85 find $\frac{1}{6}$ of 240 which is 40. Since 160 x 200 = 32,000, the more accurate approximation is 40,000. The exact answer is 40,247

Example D : $127 \times 318 =$

Note : 127 can be thought of as being 12 $\frac{1}{2}$% which is equal to $\frac{1}{8}$. To approximate 127 x 318 find $\frac{1}{8}$ of 320 which is 40. Since 100 x 300 = 30,000 the more accurate approximation is 40,000. The exact answer is 40,386.

Example E : $252 \times 363 =$

Note : 252 can be thought of as being 25% which is equal to $\frac{1}{4}$. To approximate 252 x 363 find $\frac{1}{4}$ of 360 which is 90. Since 300 x 400 = 120,000 the more accurate approximation is 90,000.

Example F: $5302 \div 11 =$

Note : 11 can be thought of as 11 $\frac{1}{9}$% which is equal to $\frac{1}{9}$.

Think of the problem as being $5300 \div \frac{1}{9}$. To quickly approximate, find 53 x 9 = 477. If you were dividing 5302 by 11, the quotient would have 3 digits, Therefore the answer is 477. The exact answer is 482.

Example F : $7488 \div 24 =$

Note : 24 can be thought of as 25% which is equal to $\frac{1}{4}$.

Think of the problem as being $7500 \div \frac{1}{4}$. To quickly approximate find 75 x 4 = 300. If you were dividing 7488 by 24, the quotient would have 3 digits. Therefore, the answer is 300. The exact answer is 312.

Example G : $41854 \div 34 =$

Note : 34 can be thought of as $33\frac{1}{3}$% which is equal to $\frac{1}{3}$.

Think of the problem as being $42000 \div \frac{1}{3}$. To quickly Approximate find 42 x 3 = 126. If you were dividing 41854 by 34 the quotient would have 4 digits. Therefore, the answer is 1260. The exact answer is 1231.

Example H : $346527 \div 831 =$

Note : 831 can be thought of as $83\frac{1}{3}$% which is $\frac{5}{6}$. Think of the problem as being $350000 \div \frac{5}{6}$. To quickly approximate, find $35 \times \frac{6}{5} = 42$. If you were dividing 346527 by 831 the quotient would have 3 digits. Therefore the answer is 420. The exact answer is 416.9579..

Multiplication by 11

Step #1 : The first digit (units digit) of number being multiplied by 11 is the first digit of the answer.

Step #2 : The sum of the tens digit and the units digit of the number will give the tens digit of the answer. If the sum is greater than or equal to 10 make sure to carryover to Step #3.

Step #3 : Repeat addition of consecutive digits (tens to hundreds, hundreds to thousands, etc.) until the final digit of the number being multiplied by 11 is reached. If necessary, add carryovers from previous steps.

Step #4 : This final digit (plus any carryover) will be the final digit of the answer.

Example A : 35 x 11 = ________

Solution :

Step #1 : The first digit of the answer is 5.
Step #2 : 3 + 5 = 8
Step #3 : The final digit is 3.

Answer : 385

Example B : 78 x 11 = ________

Solution :

Step #1 : The first digit of the answer is 8.
Step #2 : 7 + 8 = 15 (Write down the 5 and carryover the 1).
Step #3 : 7 + 1(carryover) = 8

Answer : 858

Example C : 568 x 11 = ________

Solution L

Step #1 : The first digit of the answer is 8.
Step #2 : 6 + 8 = 14 (Write down the 4 and carryover the 1)
Step #3 : 5 + 6 + 1 (carryover) = 12 (Write down the 2 and carryover the 1),
Step #4 : The final digit is 5 + 1 = 6
Answer : 6248

Multiplication by 25

Since 25% = $\frac{1}{4}$, when multiplying a number by 25, divide it by 4 and move the decimal point two places to the right.

Example A : 25 x 24 = ________

Solution : 24 ÷ 4 = 6. Moving the decimal two places to the right gives the answer : 600.

Example B : 25 x 39 = ________

Solution : 39 ÷ 4 = 9.75. Moving the decimal two places to the right gives the answer : 975.

I would like to suggest that instead of explaining the division of 39 by 4, teach your students to think of the problem is $\frac{39}{4}$ which is $9\frac{3}{4}$. $9\frac{3}{4}$ = 9.75, thus answer is 975.

Students should memorize the following which will help in converting the fractional part to a decimal.

$\frac{1}{4}$ = 25% ; $\frac{2}{4}$ = 50% ; $\frac{3}{4}$ = 75%

Example C : 25 x 21 = ________

Solution : $\frac{21}{4} = 5\frac{1}{4}$ = 5.25 and the answer is 525.

Example D : 25 x 42 = ________

Solution : $\frac{42}{4} = 10\frac{2}{4}$ = 10.50 and the answer is 1050.

Example E : 25 x 35 = ________

Solution : $\frac{35}{4} = 8\frac{3}{4}$ = 8.75 and the answer is 875.

Double and half method of multiplication

Although this shortcut can be used in other situations, I recommend that it be used when multiplying a number that ends with five by an even number.

Step #1 : Double the number ending in 5.
Step #2 : Take half of the even number.
Step #3 : Find the product of the numbers in Step #1 and Step #2.

Example A : 15 x 12 = ________

Step #1 : Double the 15. The result is 30.
Step #2 : Take one half of 12. The result is 6.
Step #3 : 30 x 6 = 180

Example B : 35 x 14 = ________

Step #1 : Double the 35. The result is 70.
Step #2 : Take one half of 14. The result is 7.
Step #3 : 70 x 7 = 490

Example C : 45 x 16 = ________

Step #1 : Double the 45. The result is 90.
Step #2 : Take one half of 16. The result is 8.
Step #3 : 90 x 8 = 720

Example D : 25 x 18 = ________

Step #1 : Double the 25. The result is 50.
Step #2 : Take one half of 18. The result is 9.
Step #3 : 50 x 9 = 450

Example E : 55 x 14 = ________

Step #1 : Double the 55. The result is 110.
Step #2 : Take one half of 14. The result is 7.
Step #3 : 110 x 7 = 770

Example F : 15 x 18 = ________

Step #1 : Double the 15. The result is 30.
Step #2 : Take one half of 18. The result is 9.
Step #3 : 30 x 9 = 270

Multiplying two numbers close to 100 (Both numbers are more than 100)

Step # 1 : Find the difference of each number and 100. Multiply the result. This product will give the first two digits of the answer (the tens and units digits). If the product is greater than one hundred, carry the hundreds digit to Step #2.

Step # 2 : Add the difference of one of the numbers and one hundred from the other number. Add any carryover from Step #1. This result will be the remaining digits of the answer.

Example A : 107 x 105 = ________

Solution :

Step #1 : (107 - 100)(105 - 100) = 7(5) = 35
Step #2 : 107 + (105 - 100) = 107 ÷ 5 = 112

Answer : 11235

Example B : 113 x 102 = ________

Solution :

Step #1 : (113 - 100)(102 - 100) = 13(2) = 26
Step #2 : 113 + (102 - 100) = 113 + 2 = 115

Answer : 11526

Example C : 106 x 109 = ________

Solution :

Step #1 : (106 - 100)(109 - 100) = 6(9) = 54
Step #2 : 106 + (109 - 100) = 106 + 9 = 115

Answer : 11554

Example D : 123 x 103 = ________

Solution :

Step #1 : (123 - 100)(103 - 100) = 23(3) = 69
Step #2 : 123 ÷ (103 - 100) = 123 + 3 = 126

Answer : 12669

Multiplying numbers whose tens digits add up to 10 and whose units digits are the same

Step #1 : Multiply the units digits together. This should give the first two digits of the answer. If the product consists of one digit, place a 0 in front when writing the answer down.

Step #2 : To the product of the tens digits add the units digit of one of the numbers.

Example A : 43 x 63 = __________

Step #1 : 3 x 3 = 09
Step #2 : 4(6) + 3 = 24 + 3 = 27

Answer : 2709

Example B : 26 x 86 = __________

Step #1 : 6 x 6 = 36
Step #2 : 2(8) + 6 = 16 + 6 = 22

Answer : 2236

Example C : 35 x 75 = __________

Step #1 : 5 x 5 = 25
Step #2 : 3(7) + 5 = 21 + 5 = 26

Answer : 2625

Example D : 17 x 97 = __________

Step #1 : 7 x 7 = 49
Step #2 : 1(9) + 7 = 9 + 7 = 16

Answer : 1649

Example E : 58 x 58 = __________

Step #1 : 8 x 8 = 64
Step #2 : 5(5) + 8 = 25 + 8 = 33

Answer : 3364

Multiplying numbers ending in 5 (where the numbers preceding the 5 are both even or both odd

Step #1 : The first two digits (tens and units digits) of the answer are always 25.
Step #2 : Take one-half of the sum of the number(s) preceding the 5. To this result, add the product of the numbers preceding the 5. This will provide the remaining digits of the answer.

Example A : 45 x 85 = ________

Step #1 : The first two digits of the answer are 25.

Step #2 : $\frac{1}{2}(4 + 8) + 4(8) = 6 + 32 = 38$

Answer : 3825

Example B : 15 x 95 = ________

Step #1 : The first two digits of the answer are 25.

Step #2 : $\frac{1}{2}(1 + 9) + 1(9) = 5 + 9 = 14$

Answer : 1425

Example C : 115 x 35 = ________

Step #1 : The first two digits of the answer are 25.

Step #2 : $\frac{1}{2}(11 + 3) + 11(3) = 7 + 33 = 40$

Answer : 4025

Example D : 25 x 65 = ________

Step #1 : The first two digits of the answer are 25.

Step #2 : $\frac{1}{2}(2 + 6) + 2(6) = 4 + 12 = 16$

Answer : 1625

Multiplying by 12

The key to this shortcut is to always remember to double and add to the right. You continue doing this until you get to the last digit (the left side of the number). At this point you imagine doubling 0 and add this last digit (plus any carryover),

Example A : 63 x 12 = __________.

Step #1 : Double the 3 and add to the right. The result is 6.
Step #2 : Double the 6 and add to the right (add it to the 3). The result is 15. Write down the 5 and carryover the 1.
Step #3 : Double 0 and add 6 plus the carryover from Step #2. the result is 7.

Answer : 756

Example B : 42 x 12 = __________

Step #1 : Double the 2 and add to the right. The result is 4.
Step #2 : Double the 4 and add to the right (add it to the 2). The Result is 10.. Write down the 0 and carryover the 1.
Step #3 : Double 0 and add 4 plus the carryover from Step #2. The result is 5.

Answer : 504

Example C : 317 x 12 = __________

Step #1 : Double the 7. The result is 14. Write down the 4 and carryover the 1 to Step #2.
Step #2 : Double the 1 and add to the right (add to the 7) plus the carryover from Step #1. The result is 10. Write down the 0 and carryover the 1 to Step #3.
Step #3 : Double the 3 and add to the right (add to the 1) plus the carryover from Step #2. The result is 8.

Step #4 : Double 0 and add 3. The result is 3.

Answer : 3804

Squaring numbers ending in 5

Step #1 : The first two digits (tens and units) will be 25.
Step #2 : The product of the tens digit and the tens digit increased by 1 will provide the final digits of the answer.

Example A : 45^2 = __________

Step #1 : The first two digits of the answer are 25.
Step #2 : $4(4 + 1) = 4(5) = 20$

Answer : 2025

Example B : 85^2 = __________

Step #1 : The first two digits of the answer is 25.
Step #2 : $8(8 + 1) = 8(9) = 72$

Answer : 7225

Example C : 105^2 = __________

Step #1 : The first two digits of the answer is 25.
Step #2 : $10(10 + 1) = 10(11) = 110$

Answer : 11025

Example D : 65^2 = __________

Step #1 : The first two digits of the answer is 25.
Step #2 : $6(6 + 1) = 6(7) = 42$

Answer : 4225

Example E : 145^2 = __________

Step #1 : The first two digits of the answer is 25.
Step #2 : $14(14 + 1) = 14(15) = 210$

Answer : 21025

Squaring any 2-digit number

Step #1 : Square the units digit. This should give the units digit of the answer. If a 2-digit number results, carryover the tens digit to Step #2.

Step #2 : Double the product of the tens digit and the units digit. Add any carryover from Step #1. Write the units digit of this result. If a 2-digit number results, carryover the tens digit to Step #3.

Step #3 : Square the tens digit. Add any carryover from Step #2. The result will be the final digits of the answer.

Example A : 32^2 = __________

Step #1 : $2^2 = 4$
Step #2 : $2(3)(2) = 12$ (Write down the 2 and carryover the 1.)

Step #3 : $3^2 + 1$ (carryover) $= 9 + 1 = 10$

Answer : 1024

Example B : 43^2 = __________

Step #1 : $3^2 = 9$
Step #2 : $2(4)(3) = 24$ (Write down the 4 and carryover the 2.)
Step #3 : $4^2 + 2$ (carryover) $= 16 + 2 = 18$

Answer : 1849

Example C : 57^2 = __________

Step #1 : $7^2 = 49$ (Write down the 9 and carryover the 4.)
Step 32 : $2(5)(7) + 4 = 70 + 4 = 74$ (Write down the 4 and carryover the 7,)
Step #3 : $5^2 + 7$ (carryover) $= 25 + 7 = 32$.

Answer : 3249

Example D : 104^2 = __________

Although the shortcut is usually used to square 2-digit numbers it can be adjusted to square 3-digit numbers.

Step #1 : $4^2 = 16$ (Write down the 6 and carryover the 1)
Step #2 : $2(10)(4) + 1$ (carryover) $= 80 + 1 = 81$ (Write down the 1 and carryover the 8)

Subtracting whole numbers

In teaching students a method for subtracting faster, I recommend that they first practice subtracting two digis at a time. The first examples shown should not require borrowing from the hundreds place.

Example A : 945 - 532 = __________

Step #1 : 45 - 32 = 13
Step #2 : 9 - 5 = 4

Answer : 413

Example B : 859 - 246 = __________

Step #1 : 59 - 46 = 13
Step #2 : 8 - 2 = 6

Answer : 613

Example C : 936 - 512 = __________

Step #1 : 36 - 12 = 24
Step #2 : 9 - 5 = 4

Answer : 424

Example D : 789 - 152 = __________

Step #1 : 89 - 52 = 37
Step #2 : 7 - 1 = 6

Answer : 637

Exqmple E : 5736 - 5221 = __________

Step #1 : 36 - 21 = 15
Step #2 : 57 - 52 = 5

Answer : 515

Example D : 6729 - 4318 = __________

Step #1 : 29 - 18 = 11
Step #2 : 67 - 43 = 24

Answer : 2411

Now students need to learn how to handle situations where borrowing from the hundreds place is required.

Example A : 7413 - 5988 = ________

Step #1: Try to find the difference of the first two digits of each number (the tens place and units place), If the number being subtracted is larger than the other number, find the difference of this number and one hundred and add this to the other number.

13 + (100 - 88) = 13 + 12 = 25

Step #2 : Find the difference of the next two digits less 1.

(74 - 59) - 1 = 15 - 1 = 14

Step #3 : Repeat Step #1 and Step #2 if necessary

Answer : 1425

Example B : 967 - 295 = ________

Step #1 : 67 + (100 - 95) = 67 + 5 = 72
Step #2 : (9 - 2) - 1 = 7 - 1 = 6

Answer : 672

Example C : 805 - 279 = ________

Step #1 : 05 + (100 - 79) = 05 + 21 = 26
Step #2 : (8 - 2) - 1 = 6 - 1 = 5

Answer : 526

Example D : 8756 - 7588 = ________

Step #1 : 56 + (100 - 88) = 56 + 12 = 68
Step #2 : (87 - 75) - 1 = 12 - 1 = 11

Answer : 1168

Example E : 7623 - 5991 = ________

Finding the average of a set of numbers

Step #1 : Find an approximate average (if possible select a multiple of 10).
Step #2 : Find the sum of the positive and negative differences of each number and your intial guess.
Step #3 : Divide the result obtained in Step #2 by the number of terms.
Step #4 : Add the result from Step #3 to your initial guess in Step #1.

Example A : Find the average of 78, 73, 65 and 72. __________

Step #1 : The average is approximately 70.
Step #2 : (78 - 70) + (73 - 70) + (65 - 70) + (72 - 70) =
8 + 3 + (- 5) + 2 = 8
Step #3 : $\frac{8}{4} = 2$
Step #4 : 70 + 2 = 72

Answer : 72

Example B : Find the average of 69, 90, 85, 92, and 79. __________

Step #1 : The average is approximately 80.
Step #2 : (69 - 80) + (90 - 80) + (85 - 80) + (92 - 80) + (79 - 80)
= (- 11) + 10 + 5 + 12 + (- 1) = 15
Step #3 : $\frac{15}{5} = 3$
Step #4 : 80 + 3 = 83

Example C : Find the average of 48, 65, 69 and 54. __________.

Step #1 : The average is approximately 60.
Step #2 : (48 - 60) + (65 - 60) + (69 - 60) + (54 - 60) =
(- 12) + 5 + 9 + (- 6) = - 4
Step #3 : $\frac{-4}{4} = -1$
Step #4 : 60 + (- 1) = 59

Example C : Find the average of 97, 100, 81, 78, 95, 86 and 86. __________

Step #1 : The average is approximately 90.
Step #2 : (97 - 90) + (100 - 90) + (81 - 90) + (78 - 90) + (95 - 90) + (86 - 90) + (86 - 90) = 7 + 10 + (- 9) + (- 12) + 5 + (- 4) + (- 4) = - 7
Step #3 : $\frac{-7}{7}$

Find the remainder when dividing by 9

Method #1 : Find the sum of the digits. Divide this sum by 9. The remainder that results is the answer :

Example A : 583 ÷ 9 has a remainder of __________.
Solution : (5 + 8 + 3) ÷ 9 = 16 ÷ 9 which is equal to 1 remainder 7. The answer is 7.

Example B : 3572 ÷ 9 has a remainder of __________.

Solution : (3 + 5 + 7 + 2) ÷ 9 = 17 ÷ 9 which is equal to 1 remainder 8. The answer is 8.

Example C : 342 ÷ 9 has a remainder of __________.

Solution : (3 + 4 + 2) ÷ 9 = 9 ÷ 9 which is equal to 1 remainder 0. The answer is 0.

Method #2 : Continue adding the digits of the number until a single digit is obtained. If the resulting digit is less than 9, this is the remainder. If the resulting digit is 9, the remainder is 0.

Example A : 834 ÷ 9 has a remainder of __________.

Solution : 8 + 3 + 4 = 15 ; 1 + 5 = 6
The remainder is 6.

Example B : 7356 ÷ 9 has a remainder of __________.

Solution : 7 + 3 + 5 + 6 = 21 ; 2 + 1 = 3
The remainder is 3.

Example C : 87542 ÷ 9 has a remainder of __________.

Solution : 8 + 7 + 5 + 4 + 2 = 26 ;
2 + 6 = 8. The remainder is 8.

Find the remainder when dividing by 11

Step #1 : Beginning with the units digit, add every other digit from left to right.
Step #2 : Find the sum of the remaining digits
Step #3 : Subtract Step #2 from Step #1

(i) If the result is less than 11, this is the remainder.

Example A : 2537 ÷ 11 has a remainder of ________.

Step #1 : 7 + 5 = 12
Step #2 : 3 + 2 = 5
Step #3 : 12 - 5 = 7

The remainder is 7.

(ii) If the result is negative, keep adding 11 until the result is positive. This is the remainder.

Example A : 4782 ÷ 11 has a remainder of ________.

Step #1 : 2 + 7 = 9
Step #2 : 8 + 4 = 12
Step #3 : 9 - 12 = -3 ; -3 + 11 = 8

The remainder is 8,

(iii) If the result is greater than 11, keep subtracting 11 until the result is less than 11.

Example A : 2819 ÷ 11 has a remainder of ________.

Step #1 : 9 + 8 = 17
Step #2 : 1 + 2 = 3
Step #3 : 17 - 3 = 14 ; 14 - 11 = 3

The remainder is 3.

(iv) If the result is 11, than the remainder is 0.

Example A : 2849 ÷ 11 has a remainder of ________.

(9 + 8) - (4 + 2) = 17 - 6 = 11 ;
The remainder is 0.

Special problems

1. $1 + 2 + 3 + 4 + \ldots + n =$ __________.

Rule : $\frac{n(n+1)}{2}$

Example A : $1 + 2 + 3 + \ldots + 10 =$ __________.

Solution : $\frac{10(10+1)}{2} = \frac{110}{2} = 55$

Example B : $1 + 2 + 3 + \ldots + 19 =$ __________.

Solution : $\frac{19(19+1)}{2} = \frac{380}{2} = 190$

Example C : $1 + 2 + 3 + \ldots + 15 =$ __________.

Solution : $\frac{15(15+1)}{2} = \frac{240}{2} = 120$

2. $2 + 4 + 6 + \ldots + n =$ __________.

Rule : $\frac{n(n+2)}{4}$

Example A : $2 + 4 + 6 + \ldots + 20 =$ __________.

Solution : $\frac{20(20+2)}{4} = \frac{20}{2}(\frac{22}{2}) = 10(11) = 110$

Example B : $2 + 4 + 6 + \ldots + 14 =$ __________.

Solution : $\frac{14(14+2)}{4} = \frac{14}{2}(\frac{16}{2}) = 7(8) = 56$

Example C : $2 + 4 + 6 + \ldots + 24 =$ __________.

Solution : $\frac{24(24+2)}{4} = \frac{24}{2}(\frac{26}{2}) = 12(13) = 15$

3. $1 + 3 + 5 + \ldots + n =$ __________.

Step #2 : 4(100 - 1) = 400 - 4 = 396

Example C : 932 - 239 = ________

Step #1 : 9 - 2 = 7
Step #2 : 7(100 - 1) = 700 - 7 = 693

Example D : 542 - 245 = ________

Step #1 : 5 - 2 = 3
Step #2 : 3(100 - 1) = 300 - 3 = 297

Example E : 852 - 258 = ________

Step #1 : 8 - 2 = 6
Step #2 : 6(100 - 1) = 600 - 6 = 594

5. Subtraction (Interchange in groups of two)

Example A : 3423 - 2334 = ________.

Step #1 : Find the difference between the 2-digit number formed by the thousands and hundreds digits and the 2-digit number formed by the tens and units digits.

34 - 23 = 11

Step #2 : Multiply the result of Step #1 by 99. This will give the answer. Hint : Multiply the result of Step #1 by 100 - 1.

11(100 - 1) = 1100 - 11 = 1089

Example B : 3112 - 1231 = ________.

Step #1 : 31 - 12 = 19
Step #2 : 19(100 - 1) = 1900 - 19 = 1881

Example C : 5314 - 1453 = ________.

Step #1 : 53 - 14 = 39
Step #2 : 39(100 - 1) = 3900 - 39 = 3861

Example D : 4735 - 3547 = ________.

Step #1 : 47 - 35 = 12
Step #2 : 12(100 - 1) = 1200 - 12 = 1188

Example E : 7857 - 5778 = __________.

Step #1 : 78 - 57 = 21
Step #2 : 21(100 - 1) = 2100 - 21 = 2079

Example F : 3952 - 5239 = __________.

Step #1 : 39 - 52 = - 13

Step #2 : - 13(100 - 1) = - 1300 + 13 = - 1287

Example G : 4361 - 6143 = __________.

Step #1 : 43 - 61 = - 18
Step #2 : - 18(100 - 1) = - 1800 ÷ 18 = - 1782

6. 421 ÷ 9 = ________________ (mixed number)

Step #1 : Find the sum of the digits of the given number. This will be the numerator of a fraction whose denominator is 9. If the result is a proper fraction write this as part of the answer. If the result is an improper fraction, convert it into a mixed number ; write down the fractional part, then carry the whole number part to Step #2.

$$\frac{4 + 2 + 1}{9} = \frac{7}{9}$$

Step #2 : Find the sum of the hundreds digit and the tens digit (plus any carryover from Step #1). This is the units digit of the answer.

(4 + 2) + 0 = 6

Step #3 : The hundreds digit of the number being divided by 9 (plus any carryover from Step #2) will be the tens digit of the answer.

4 + 0 = 4

Answer : $46\frac{7}{9}$

14. The GCF of 12 and 20 is __________.

Step #1 : If necessary, double or triple or quadruple, or ... the smaller number. The purpose of this is to make the difference between the two numbers a minimum.

Step #2 : Find the difference of the result of Step #1 and the other number. This difference may be the GCF of the two given numbers. To find out, try dividing each of the given numbers by this difference. If not, it is a factor of one of these numbers.

Solution : 2(12) - 20 = 24 - 20 = 4

Answer : 4

Example A : The GCF of 24 and 78 is __________.

78 - 3(24) = 78 - 72 = 6 .

Answer : 6

Example B : The GCF of 21 and 56 is __________.

3(21) - 56 = 63 - 56 = 7

Answer : 7

Example C : The GCF of 20 and 68 is __________

68 - 3(20) = 68 - 60 = 8

Note : Neither 20 or 68 can be divided by 8. The only factor of 8 that can divided both numbers is 4. Thus, the GCF of 20 and 68 is 4.

Example D : The GCF of 15 and 51 is __________.

51 - 3(15) = 51 - 45 = 6

Note : Neither 15 or 51 can be divided by 6. The Only factor of 6 that can divide both numbers is 3. Thus, the GCF of 15 and 51 is 3.

Answer : 3

15. The LCM of 20 and 50 is __________.

Step #1 : Find the GCF of 20 and 50. Use the trick above for finding the GCF.

$50 - 2(20) = 50 - 40 = 10$

Step #2 : Divide the product of the two numbers by the result of Step #1.

$\frac{20(50)}{10} = \frac{20}{10} \text{ x } 50 = 2(50) = 100$

Answer : 100

Example A : The LCM of 18 and 45 is __________.

Step #1 : Find the GCF. $45 - 2(18) = 45 - 36 = 9$

Step #2 : $\frac{18(45)}{9} = \frac{18}{9}(45) = 2(45) - 90$

Answer : 90

Example B : The LCM of 21 and 35.

Step #1 : The GCF of 21 and 35 is 7.

Step #2 : $\frac{21(35)}{7} = \frac{21}{7}(35) = 3(35) = 105$

Answer : 105

Example C : The LCM of 12 and 40 is __________.

Step #1 : The GCF of 12 and 40 is 4.

Step #2 : $\frac{12(40)}{4} = \frac{12}{4}(40) = 3(40) = 120$

ANSWER : 120

16. The LCM of 12, 18, and 20 is __________.

Step #1 : Find the GCD of the first two numbers.
The GCD of 12 and 18 is 6.

Step #2 : To find the LCM of the first wo numbers, find the product

21. Multiplying mixed numbers where the fractional part add up to 1 and the whole number parts are the same.

Step #1 : Multiply the fractional parts. Write down the result.
Step #2 : Find the product of of one of the whole numbers and that whole number increased by 1.

Example A : $6\frac{2}{3} \times 6\frac{1}{3} =$ __________ (mixed number)

Step #1 : $\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$
Step #2 : $6(6 + 1) = 6(7) = 42$

Answer : $42\frac{2}{9}$

Example B : $9\frac{3}{5} \times 9\frac{2}{5} =$ __________ (mixed number)

Step #1 : $\frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$
Step #2 : $9(9 + 1) = 9(10) = 90$

Answer : $90\frac{6}{25}$

Example C : $5\frac{4}{7} \times 5\frac{3}{7} =$ __________ (mixed number)

Step #1 : $\frac{4}{7} \times \frac{3}{7} = \frac{12}{49}$
Step #2 : $5(5 + 1) = 5(6) = 30$

Answer : $30\frac{12}{49}$

22. Multiplying mixed numbers where the fractional part are the same.

Before doing the shortcut, check to see that the result of taking the fractional part of the sum of the whole numbers is a whole number.

Solution : $\frac{15(42)}{35} = \frac{15}{5}(\frac{42}{7}) = 3(6) = 18$

25. Triangular numbers : 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136, 153, 171, 190, 210

Rule : $\frac{n(n-1)}{2}$

Example A : Find the 9^{th} triangular number. ________.

Solution : $\frac{9(9+1)}{2} = \frac{9(10)}{2} = 45$

Example B : Find the 6^{th} triangular number. ________.

Solution : $\frac{6(6+1)}{2} = \frac{6(7)}{2} = 21$

26. The median of 2, 3, 5, 7, 3, 5, 3 & 7 is ________.

If you are given an odd number of terms, the median is the number in the middle when the terms are arranged in either ascending order or descending order.

Example A : The median of 5, 13, 7, 10, and 4 is ________.

Solution : Arrange the terms in ascending order : 4, 5, 7, 10. and 13. Select the number in the middle.
Answer : 7

If you are given an even number of terms, the median is the average of the Two numbers in the middle when the terms are arranged in either ascending order or descending order.

Example A : The median of 14, 6, 10, 24, 2 and 18.

Solution : Arrange the terms in ascending order : 2, 6, 10, 14, 18, and 24. Find the average of the two middle terms ($\frac{10+14}{2} = 12$). Answer : 12

36. The number of subsets of $\{1, 3, 5, 7, 9\}$ is ________.

Rule : 2^n , where n is the number of elements in the set.

Solution : $2^5 = 32$

Example A : How many subsets does a 4-element set have ? ________.

Solution : $2^4 = 16$

Example B : The number of subsets of $\{a, b, c\}$ is ________.

Solution : $2^3 = 8$

37. Set A = {a, b, c, d}. How many proper subsets does set A have ?

A set has 2^n subsets. Since the set itself is considered to be improper, the number of proper subsets it has is $2^n - 1$.

Solution : has $2^3 - 1 = 8 - 1 = 7$

Example A : How many proper subsets does a 6-element set have ?

Solution : $2^6 - 1 = 64 - 1 = 63$

Example B : Set B = (3, 8, 0, 5, 2}. How many proper subsets does set B have?

Solution : $2^5 - 1 = 32 - 1 = 31$

Note : A set has exactly one improper subset (which is the set itself).

Example A : How many improper subsets does a 3-element set have ?

Solution : 1

38. The number of elements in the Power Set of {M, A, T, H} is ________.

The power set of a set is a set whose elements are the subsets of the set.

Rule : 2^n , where n equals the number of elements in the set.

Example A : A triangle has integral sides of 8, 14, and x. The smallest value of x is ________.

Solution : $|8 - 14| + 1 = 6 + 1 = 7$.

Example B : A triangle has integral sides of 5, 17, and x. The smallest value of x is ________.

Solution : $|5 - 17| + 1 = 12 + 1 = 13$

67. A pair of dice is thrown. Find the probability that the sum of the faces landing up is 5. ________

When a pair of dice are thrown there are 36 possible outcomes. If the sum of the faces is 7 or less, the number of possible outcomes is 1 less than the sum. If the sum of the faces is 7 or more, the sum of the number of possible of outcomes and the sum is equal to 13.

Solution : Since 5 is less than 7, there are 4 possible outcomes. The probability is equal to $\frac{4}{36}$ which is equal to $\frac{1}{9}$.

Example A : A pair of dice is thrown. Find the probability that the sum of the faces landing up is 10. ________

Solution : Since 10 is more than 7, the number of possible outcomes is 3 (Note : $10 + 3 = 13$). The probability Is $\frac{3}{36}$ which is equal to $\frac{1}{12}$.

Example B : A pair of dice is thrown. Find the probability that the sum of the Faces landing up is 4.

Solution : Since 4 is less than 7, there are 3 possible outcomes. The probability is $\frac{3}{36}$ which is equal to $\frac{1}{12}$.

68. A pair of dice is thrown. The probability that the sum is divisible by a is ________.

Solution : If $2 \leq a \leq 6$, the probability is equal to the reciprocal of a, except if $a = 5$ then answer is $\frac{7}{36}$.

CHANGING FROM A BASE TO BASE 10

234 base 5 equals __________ base 10.

Step #1 : Multiply the first digit to the left by the base
2(5) = 10
Step #2 : Add the middle digit with the result from Step #1.
4 + 10 = 13
Step #3 : Multiply the result from Step #2 by the base.
13(5) = 65
Step #4 : Add the digit to the right by the result from Step #3.
4 + 65 = 69

Example A : 213 base 8 equals __________ base 10.

Step #1 : 2(8) = 16
Step #2 : 1 + 16 = 17
Step #3 : 17(8) = 136
Step #4 : 3 + 136 = 139

Example B : 321 base 4 equals __________ base 10.

Step #1 : 3(4) = 12
Step #2 : 2 + 12 = 14
Step #3 : 14(4) = 56
Step #4 : 56 + 1 = 57

Example C : 312 base 5 equals __________ base 10.

Step #1 : 3(5) = 15
Step #2 : 1 + 15 = 16
Step #3 : 16(5) = 80
Step #4 : 80 + 2 = 82

Example D : 142 base 6 equals __________ base 10.

Step #1 : 1(6) = 6
Step #2 : 4 + 6 = 10
Step #3 : 10(6) = 60
Step #4 : 60 + 2 = 62